Int. J. Heat Mass Transfer. Vol. 27, No. 6, pp. 851–855, 1984
Printed in Great Britain

0017-9310/84 $3.00 + 0.00
Pergamon Press Ltd.

A MATHEMATICAL ANALYSIS OF PNEUMATIC DRYING OF GRAINS—II. FALLING RATE DRYING

SHIGERU MATSUMOTO* and DAVID C. T. PEI
Department of Chemical Engineering, University of Waterloo, Waterloo, Ontario N2L 3G1, Canada

(*Received* 5 *February* 1983 *and in revised form* 15 *June* 1983)

Abstract—Analytical solutions, based on the diffusion equation, are presented in describing the pneumatic drying of grains during the falling rate period. Humidity variation of the air along the length of the duct is taken into consideration as well as the internal and external resistance of the mass transfer in the grain. For a Biot number greater than 100, the internal diffusion resistance is the dominant factor for the drying process.

NOMENCLATURE

a	grain radius [m]
a_v	surface area of solids per unit dryer volume [m^{-1}]
B	mass transfer Biot number, ka/D_m
C	moisture concentration of solid [kg m^{-3}]
D_m	moisture diffusion coefficient [m^2 s^{-1}]
G_a, G_s	dry air and solids flow rate [kg s^{-1}]
H	absolute humidity
k	mass transfer coefficient based on the moisture concentration driving force [m s^{-1}]
M	parameter, $1/(3m\alpha)$
m	mass flow ratio, G_s/G_a
N	dimensionless drying rate, $R_d a/[\rho_s(W_0 - W_{eq,0})]$
R	dimensionless radius, r/a
R_d	drying rate [kg m^{-2} s^{-1}]
r	radial coordinate [m]
S	cross sectional area of dryer [m^2]
t	time [s]
u_s	solid velocity [m s^{-1}]
W	moisture content
x	distance [m]
Y	$(W_0 - W)/(W_0 - W_{eq,0})$
Z	$(W_0 - W_{eq})/(W_0 - W_{eq,0})$.

Greek symbols

α	equilibrium constant, equation (1)
β	constant, equation (1)
θ	Fourier number, $D_m t/a^2$
ξ_n	eigen values
ρ_s	solid density [kg m^{-3}].

Subscripts

av	average
eq	equilibrium
0	initial.

1. INTRODUCTION

IN PART I [1], the constant drying rate was assumed in the mathematical model for pneumatic drying of grains. However, as drying progresses and the fraction of the surface wetted decreases, the rate of drying falls as the moisture content of the solid drops. The mechanism of moisture movement within the solids during the falling rate period is a very complicated one, depending on the materials to be dried. For the analysis of drying of usual grains such as cereal grains, it has been found that the diffusion equation for spheres describes very well the drying behavior in the early stages of drying during the falling rate period. Therefore, a number of theoretical and experimental investigations on the drying curve of grain have been based on the diffusion equation [2–5]. Most of them assumed that the resistance of mass transfer existed only within a grain, and that the humidity of the drying air was maintained constant. In the pneumatic drying process, however, humidity of the surrounding air progressively increases along the duct length as noted in the preceding paper [1].

The purpose of this paper is to establish analytical solutions, on the basis of the diffusion equation, describing the rate of drying and the moisture distribution of grain in such a situation. The resistance of mass transfer in the convective layer of air at the outer surface of the grain is also taken into account as well as the resistance within the grain.

2. MATHEMATICAL MODEL

The model to be considered is based on the following assumptions:

(1) The internal moisture transfer during drying can be expressed by $J = -D_m(\partial C/\partial r)$ where the diffusion coefficient D_m is assumed to be constant.

(2) Due to the short residence time in pneumatic drying, the solid temperature is assumed to be constant.

(3) Equilibrium moisture content is related to the humidity of drying air as a linear function

$$W_{eq} = \alpha H + \beta. \quad (1)$$

This is justified for cereal grains, such as corn and wheat, in the humidity range considered in the pneumatic drying process (Fig. 1).

The diffusion equation of moisture within a sphere is

$$\frac{\partial C}{\partial t} = D_m\left(\frac{\partial^2 C}{\partial r^2} + \frac{2}{r}\frac{\partial C}{\partial r}\right) \quad \text{in} \quad 0 \leqslant r \leqslant a,\ t > 0. \quad (2)$$

* Present address: Department of Chemical Engineering, Tohoku University, Aoba, Sendai 980, Japan.

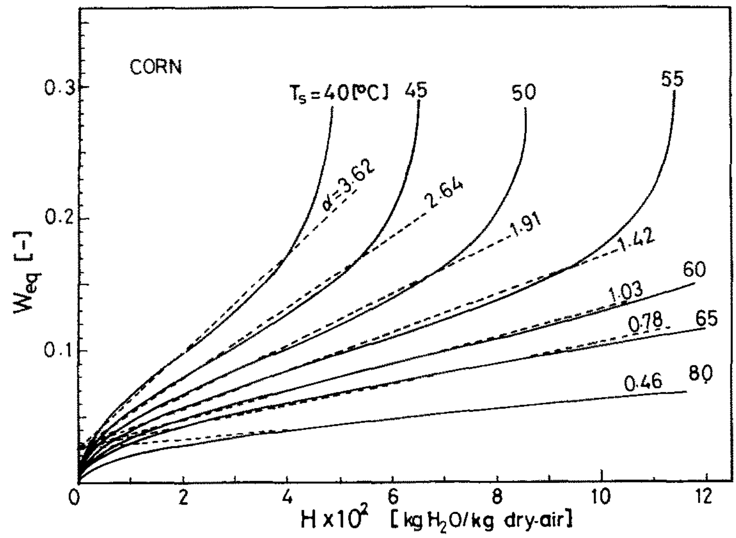

FIG. 1. Equilibrium moisture content of corn as a function of the humidity of air.

Initial and boundary conditions are

$$C = C_0 \quad \text{in} \quad 0 \leqslant r \leqslant a,\ t = 0, \tag{3}$$

$$R_d = -D_m \frac{\partial C}{\partial r} = k(C - C_{eq}) \quad \text{at} \quad r = a,\ t > 0. \tag{4}$$

The humidity change of air along the duct length is expressed by the following mass balance equation

$$G_a \frac{dH}{dx} = (a_v S) R_d \quad \text{in} \quad x > 0,$$

and (5)

$$H = H_0 \quad \text{at} \quad x = 0.$$

Rearranging equation (5) using the relation $dx/dt = u_s$, the variation rate of the humidity of ambient air is

$$\frac{dH}{dt} = \left(\frac{a_v S u_s}{G_a}\right) R_d = \left(\frac{3m}{a\rho_s}\right) R_d. \tag{6}$$

From equations (4) and (6), the relationships on the drying rate are

$$R_d = -D_m \left(\frac{\partial C}{\partial r}\right)_{r=a} = k(C_{r=a} - C_{eq}) = \left(\frac{a\rho_s}{3m}\right) \frac{dH}{dt}. \tag{7}$$

The above equations can be rewritten in terms of the moisture content as

$$\frac{\partial W}{\partial t} = D_m \left(\frac{\partial^2 W}{\partial r^2} + \frac{2}{r} \frac{\partial W}{\partial r}\right) \quad \text{in} \quad 0 \leqslant r \leqslant a,\ t > 0, \tag{8}$$

$$W = W_0 \quad \text{in} \quad 0 \leqslant r \leqslant a, \quad \text{and} \quad H = H_0; \quad t = 0, \tag{9}$$

$$-D_m \left(\frac{\partial W}{\partial r}\right)_{r=a} = k(W_{r=a} - W_{eq})$$

$$= \left(\frac{a}{3m}\right) \frac{dH}{dt}, \quad t > 0. \tag{10}$$

By using equation (1), equations (9) and (10) are rewritten as

$$W = W_0 \quad \text{in} \quad 0 \leqslant r \leqslant a,$$

$$\text{and} \quad W_{eq} = W_{eq,0}; \quad t = 0, \tag{11}$$

$$-D_m \left(\frac{\partial W}{\partial r}\right)_{r=a} = k(W_{r=a} - W_{eq}) = \left(\frac{a}{3m\alpha}\right) \frac{dW_{eq}}{dt}. \tag{12}$$

Introducing the following dimensionless variables

$$Y = \frac{W_0 - W}{W_0 - W_{eq,0}}, \quad Z = \frac{W_0 - W_{eq}}{W_0 - W_{eq,0}},$$

$$R = \frac{r}{a}, \quad \theta = \frac{D_m t}{a^2}, \tag{13}$$

the above system becomes

$$\frac{\partial Y}{\partial \theta} = \frac{\partial^2 Y}{\partial R^2} + \frac{2}{R} \frac{\partial Y}{\partial R} \quad \text{in} \quad 0 \leqslant R \leqslant 1,\ \theta > 0, \tag{14}$$

$$Y = 0 \quad \text{in} \quad 0 \leqslant R \leqslant 1, \quad \text{and} \quad Z = 1, \theta = 0, \tag{15}$$

$$-\left(\frac{\partial Y}{\partial \theta}\right)_{R=1} = B(Y_{R=1} - Z) = M \frac{dZ}{d\theta}, \quad \theta > 0, \tag{16}$$

where

$$B = ka/D_m \quad \text{and} \quad M = 1/(3m\alpha). \tag{17}$$

There are two parameters, B and M, to be considered. The Biot number represents the ratio of the external mass transfer rate to the internal diffusion rate. M is a unique parameter for the pneumatic drying system. It represents the capacity ratio of air to solids.

3. SOLUTIONS AND ANALYSIS

Solutions of the above equations can be obtained by Laplace transforms

$$Y(\theta, R) = \frac{3M}{3M+1} + \frac{2M}{R} \sum_{n=1}^{\infty} \frac{1 - (M/B)\xi_n^2}{Q_n} \frac{\sin \xi_n R}{\sin \xi_n} \exp(-\xi_n^2 \theta), \tag{18}$$

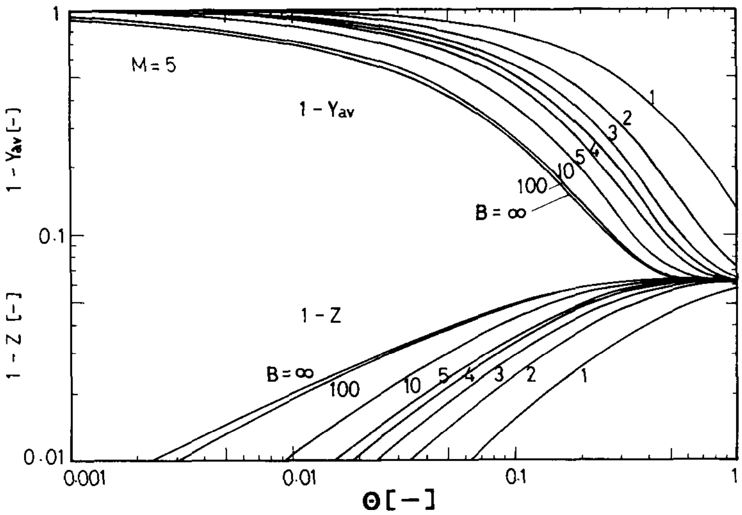

FIG. 2. Variations of average moisture content of solids and humidity of air with time.

and

$$Z(\theta) = \frac{3M}{3M+1} + 2M \sum_{n=1}^{\infty} \exp(-\xi_n^2\theta)/Q_n, \quad (19)$$

where

$$Q_n = (M/B)^2\xi_n^4 + M[M-(M/B)-(2/B)]\xi_n^2 + 3M + 1, \quad (20)$$

and ξ_n are the positive roots of

$$\tan \xi = \xi[1-(M/B)\xi^2]/\{1-M[(1/B)-1]\xi^2\}. \quad (21)$$

The dimensionless average moisture content within the sphere is

$$Y_{av}(\theta) = 3M/(3M+1) - 6M^2 \sum_{n=1}^{\infty} \exp(-\xi_n^2\theta)/Q_n, \quad (22)$$

and the dimensionless rate of drying is

$$N(\theta) = R_d/[D_m\rho_s(W_0 - W_{eq,0})/a]$$

$$= 2M^2 \sum_{n=1}^{\infty} \xi_n^2 \exp(-\xi_n^2\theta)/Q_n. \quad (23)$$

If the value of B approaches infinity (the external mass transfer coefficient is very large, $k \to \infty$, or the diffusion coefficient is very small, $D_m \to 0$), then the above solution becomes

$$Y(\theta, R) = \frac{3M}{3M+1} + \frac{2M}{R} \sum_{n=1}^{\infty} \frac{1}{M^2\xi_n^2 + 3M + 1} \times \frac{\sin \xi_n R}{\sin \xi_n} \exp(-\xi_n^2\theta), \quad (24)$$

$$Z(\theta) = \frac{3M}{3M+1} + 2M \sum_{n=1}^{\infty} \exp(-\xi_n^2\theta)/[M^2\xi_n^2 + 3M + 1], \quad (25)$$

$$Y_{av}(\theta) = \frac{3M}{3M+1} - 6M^2 \sum_{n=1}^{\infty} \exp(-\xi_n^2\theta)/[M^2\xi_n^2 + 3M + 1], \quad (26)$$

and

$$N(\theta) = 2M^2 \sum_{n=1}^{\infty} \xi_n^2 \exp(-\xi_n^2\theta)/[M^2\xi_n^2 + 3M + 1], \quad (27)$$

where ξ_n are the positive roots of

$$\tan \xi = \xi/(1 + M\xi^2). \quad (28)$$

These results should become identical with those obtained with the boundary condition; $Y(\theta, 1) = Z(\theta)$ and $-[\partial Y(\theta, 1)/\partial R] = M[dZ(\theta)/d\theta]$, which means that the surface moisture content of the solid is always in equilibrium with the surrounding humidity. In the stationary state ($\theta \to \infty$), the series in the above equations are equal to zero, and in this case drying no longer proceeds due to saturation. The final minimum moisture content thus becomes

$$Y(\infty, R) = Y_{av}(\infty) = Z(\infty) = 3M/(3M+1). \quad (29)$$

4. RESULTS AND DISCUSSION

Variations of the average moisture content and the humidity of the drying air are shown in Fig. 2 as a function of Fourier number with parameters of B for $M = 5$. It can be seen that the larger the value of B, the faster the drying proceeds and hence the air humidity increases rapidly. For $B > 100$, the variation of the average moisture content agrees nearly with the curve for $B = \infty$. In these cases, the saturation occurs at around $\theta = 0.6$. The saturation moisture content depends on the magnitude of M; that is, the smaller the value of M, the higher the moisture content at saturation. In Fig. 3, dimensionless drying rates are shown as a function of Fourier number. As expected, at a low Fourier number, the drying rate is significantly higher with larger values of B. However, as the Fourier number increases, it falls rapidly since the driving force of mass transfer decreases due to saturation. The drying rate also approaches to that with $B = \infty$ as B exceeds

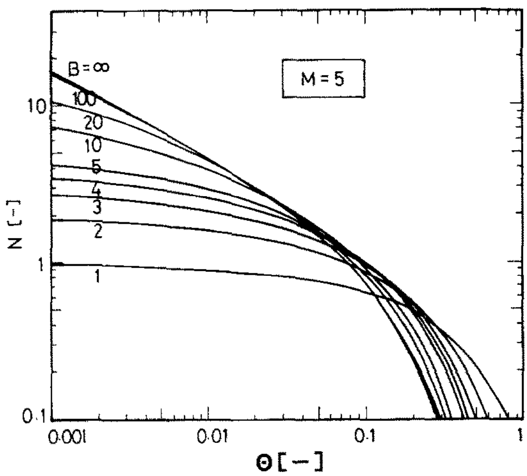

FIG. 3. Effect of B on the drying rate.

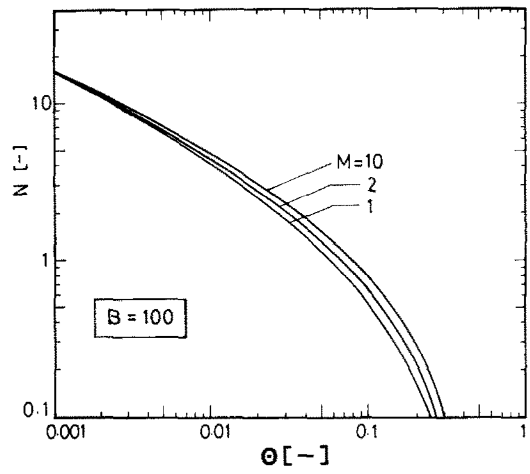

FIG. 4. Effect of M on the drying rate.

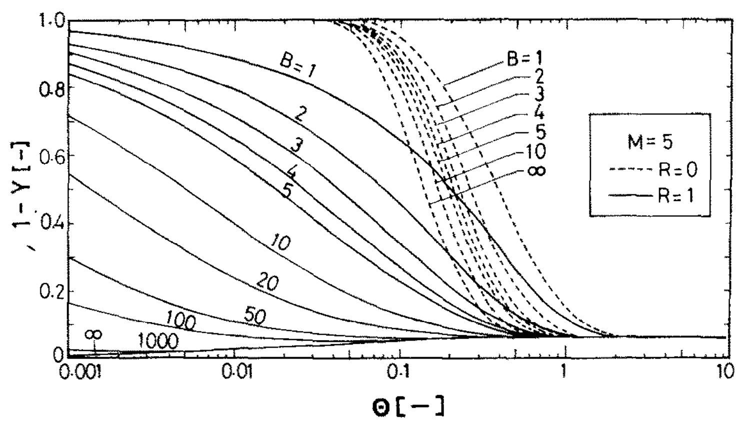

FIG. 5. Effect of B on the moisture content at the surface and center of the grain. (The solid lines represent the values at the surface, and the dotted lines at the center.)

100. The effect of the other parameter M on the drying rate is shown in Fig. 4. It appears that the rate of drying is not dependent on M. A slight increase in drying rate with higher M is considered to be caused by a larger driving force of mass transfer due to a smaller increase in the air humidity. Figure 5 shows variations of moisture content at the center and the surface of the sphere with parameters of B. It is expected that the moisture distribution in a grain is steep when B is large. For $B > 100$, the surface moisture content is very close to the case where $B = \infty$. It increases slightly with time since the equilibrium value increases with increasing air humidity. This is especially true when the mass flow ratio is large.

5. CONCLUSIONS

Based on the analytical solutions of the diffusion equation of the grain, it can be concluded that for values of $B > 100$, the internal diffusion within the grain controls the drying rate. This results in the fact that the surface moisture content will be nearly in equilibrium with the humidity of the ambient air at any time. Therefore, lower loading ratios should be recommended in drying the grains. However, it should be pointed out here that this analysis can be adopted to predict the drying curve of any material with known equilibrium data and moisture diffusion coefficient during the falling rate period.

REFERENCES

1. S. Matsumoto and D. C. T. Pei, A mathematical analysis of pneumatic drying of grains—I. Constant drying rate, *Int. J. Heat Mass Transfer* **27**, 843–849 (1984).
2. H. A. Becker, A study of diffusion in solids of arbitrary shape, with application to the drying of the wheat kernel, *J. Appl. Polymer Sci.* **1**, 212–226 (1959).
3. A. Hustrulid and A. M. Flikke, Theoretical drying curve for shelled corn, *Trans. Am. Soc. Mech. Engrs* **2**, 112–114 (1959).
4. S. Pabis and S. M. Henderson, Grain drying theory—II. A critical analysis of the drying curve for shelled maize, *J. Agric. Engng Res.* **6**, 272–277 (1961).
5. S. T. Chu and A. Hustrulid, Numerical solution of diffusion equations, *Trans. Am. Soc. Mech. Engrs* **11**, 705–708 (1968).

UNE ANALYSE MATHEMATIQUE DU SECHAGE PNEUMATIQUE DES GRAINS—II. SECHAGE A VITESSE DECROISSANTE

Résumé—Des solutions analytiques, basées sur l'équation de diffusion, sont présentées pour décrire le séchage pneumatique des grains pendant la période de décroissance du séchage. La variation d'humidité de l'air dans la longueur du tube est prise en considération aussi bien que la résistance interne et externe du transfert massique dans le grain. Pour un nombre de Biot supérieur à 100, la résistance interne de diffusion est le facteur dominant pour le mécanisme de séchage.

MATHEMATISCHE UNTERSUCHUNG DER LUFTTROCKNUNG VON GETREIDE—II. ABNEHMENDE TROCKNUNGSGESCHWINDIGKEIT

Zusammenfassung—Analytische Lösungen auf der Grundlage der Diffusionsgleichung werden für die pneumatische Trocknung von Getreide während des zweiten Trocknungsabschnitts (abnehmende Trocknungsgeschwindigkeit) angegeben. Sowohl die Änderung der Luftfeuchtigkeit entlang des Kanals als auch der innere und äußere Stoffübergangswiderstand des Getreides sind berücksichtigt. Der innere Diffusions-Widerstand ist für Biot-Zahlen größer als 100 die bestimmende Einflußgröße des Trocknungsprozesses.

МАТЕМАТИЧЕСКИЙ АНАЛИЗ ПНЕВМОГАЗОВОЙ СУШКИ ЗЕРНА—II. ПЕРИОД ПАДАЮЩЕЙ СКОРОСТИ СУШКИ

Аннотация—Предствлены аналитические решения на основе уравнения диффузии для описания пневмогазовой сушки зерна в период падающей скорости. Учитываются изменение влажности воздуха по длине трубы-сушилки, а также внутреннее и внешнее сопротивление переносу массы в зерне. Показано, что при числе Био, превышающем 100, превалирующее влияние на процесс сушки оказывает внутреннее сопротвление диффузии влаги.